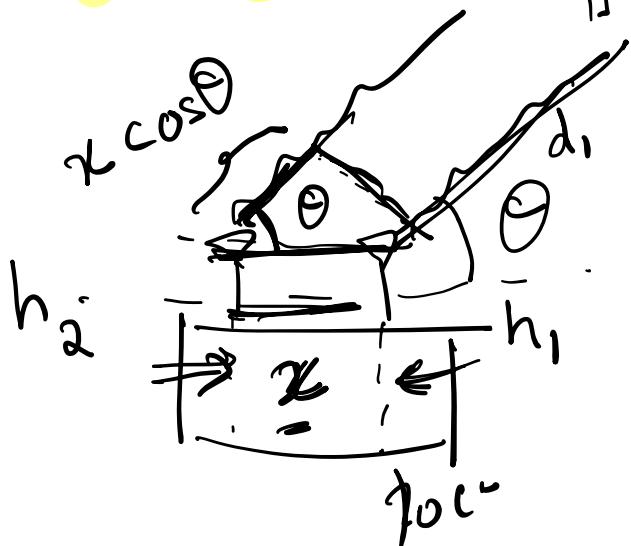


Today

- Angle of Arrival
- Multipath Profile
- Antenna Array
- Time-of-flight
- Distance & Chronos .

Angle of Arrival



$$\underline{h}_1 = -\frac{2\pi}{\lambda} d_1 \bmod 2\pi$$

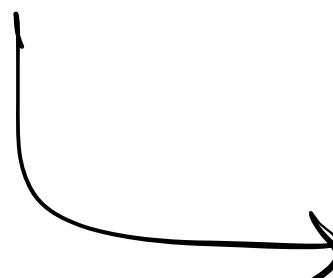
$$\underline{h}_2 = -\frac{2\pi}{\lambda} d_2 \bmod 2\pi$$

$$(d_2 - d_1) = x \cos \theta$$

$$\underline{h}_1 - \underline{h}_2 = \frac{2\pi}{\lambda} (d_2 - d_1)$$

$$= \frac{2\pi}{\lambda} x \cos \theta$$

$$\cos \theta = \frac{1}{\frac{2\pi}{\lambda}} (\underline{h}_1 - \underline{h}_2)$$



do not capture multipath.

$$(\underline{h}_1 - \underline{h}_2) = \left(\frac{2\pi}{\lambda} x \cos \theta \right) \bmod 2\pi$$

$\bmod 2\pi$

$$\cos \theta = \frac{d}{\frac{\lambda}{2}} \quad -\pi \underbrace{\qquad}_{\pi} \quad -2\pi, 2\pi$$

$$-\pi < \frac{2\pi}{\lambda} x \cos \theta < \pi$$

$\frac{2\pi}{\lambda} x$

$$-\frac{2\pi}{\lambda} x = -\pi$$

$$x = \frac{\pi}{\frac{2\pi}{\lambda}}$$

$$\frac{2\pi}{\lambda} x = \pi$$

$$x = \frac{\lambda}{2}$$



$$d_2 + d_3 > d_1$$

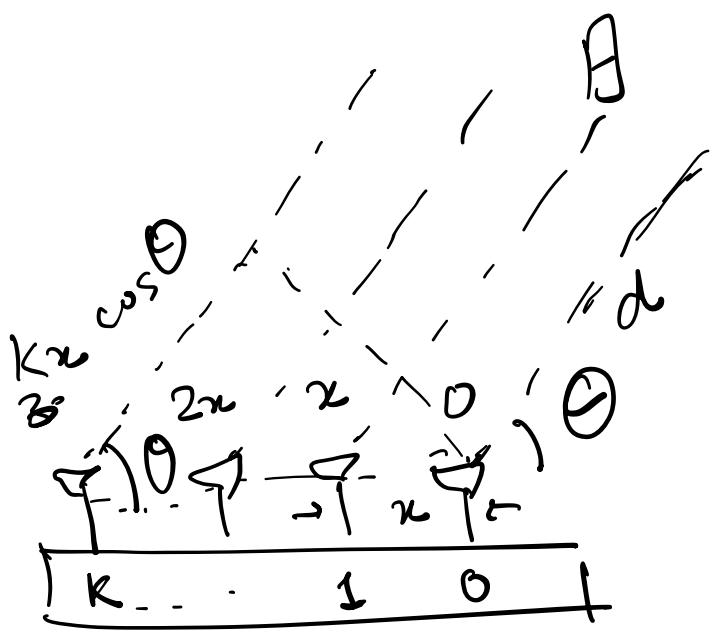
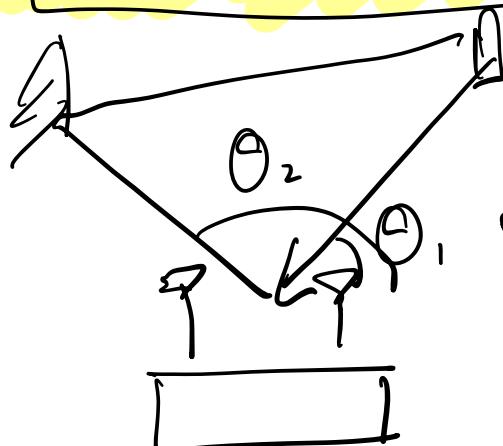
$$x = \lambda$$

$$2.4 \text{ GHz} \rightarrow 12 \text{ cm}$$

$\lambda = 12 \text{ cm}$

$$5 \text{ GHz} \xrightarrow{6c} 3 \text{ cm}$$

Multipath Profile



Uniformly Antenna or rays

$$h_k = C e^{-j \frac{2\pi}{\lambda} (kx \cos \theta + d)}$$

$$h_i = C e^{-j \frac{2\pi}{\lambda} (x \cos \theta + d)}$$

$$h_2 = C e^{-j \frac{2\pi}{\lambda} (k^2 x) \cos \Theta + d}$$

$$h_K = C e^{-j \frac{2\pi}{\lambda} (k x \cos \Theta + d)}$$

$$\underline{\underline{h}}_{1K} = \sum_{i=0}^{N-1} c_i \left(e^{-j \frac{2\pi}{\lambda} (k x \cos \theta_i + d_i)} \right)$$

10 antennas, 3 paths.

$$P(\theta) = \sum_K h_K e^{+j \frac{2\pi}{\lambda} (k x \cos \theta + d)}$$

$$h_K = c_i e^{-j \frac{2\pi}{\lambda} (k x \cos \theta_i + d_i)}$$

$$P(\theta) = \sum_{k=1}^K c_k e^{-j \frac{2\pi}{\lambda} (kx \cos \theta, d_i)}$$

$$e^{j \frac{2\pi}{\lambda} kx \cos \theta}$$

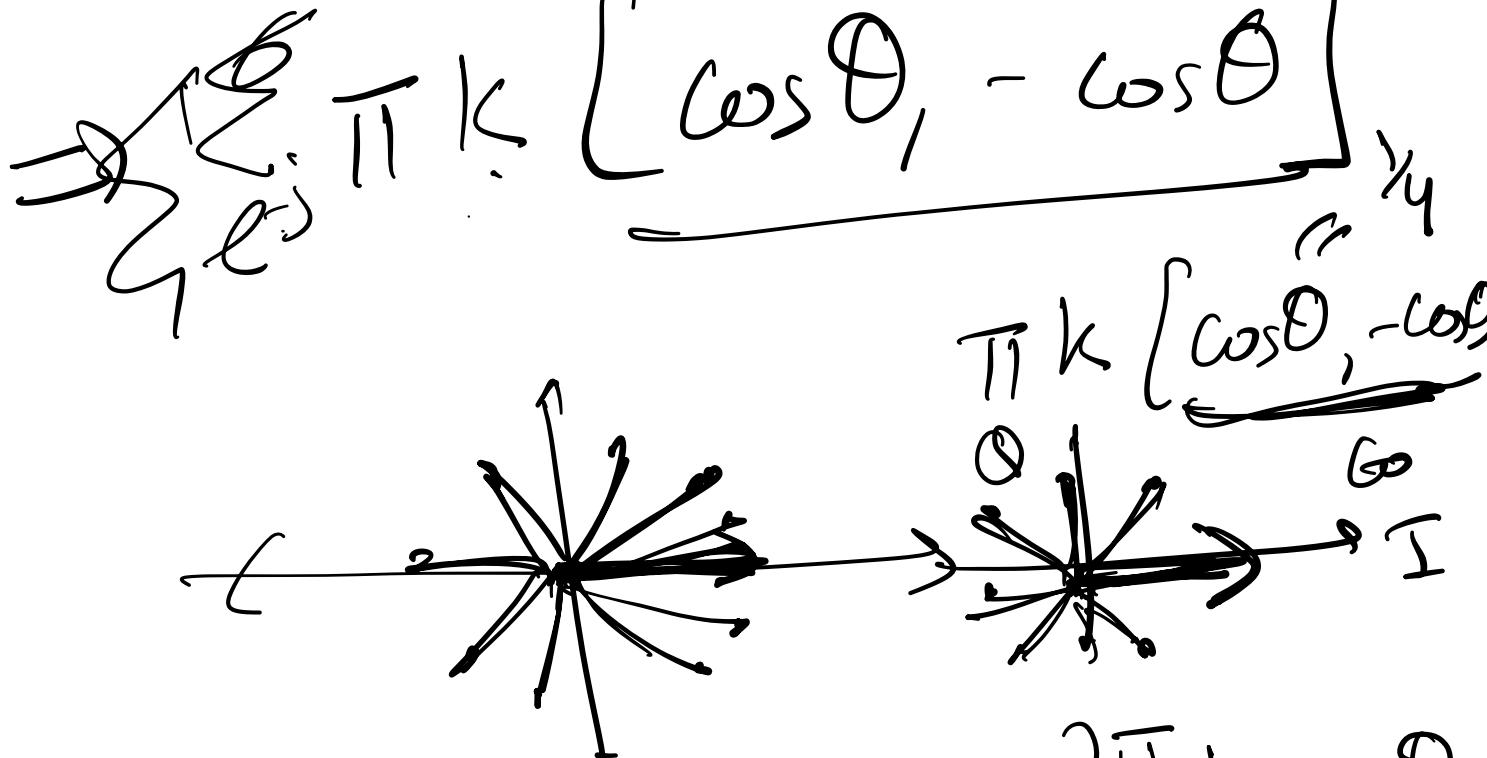
$$= C_1 e^{-j \frac{2\pi}{\lambda} d_i} \sum_k c_k e^{-j \frac{2\pi}{\lambda} kx [\cos \theta, -\cos \theta]}$$

$$\theta_i = \theta \Rightarrow \cos \theta_i = \cos \theta \Rightarrow 0$$

$$P(\theta) = K c_1 e^{-j \frac{2\pi}{\lambda} d_i}$$

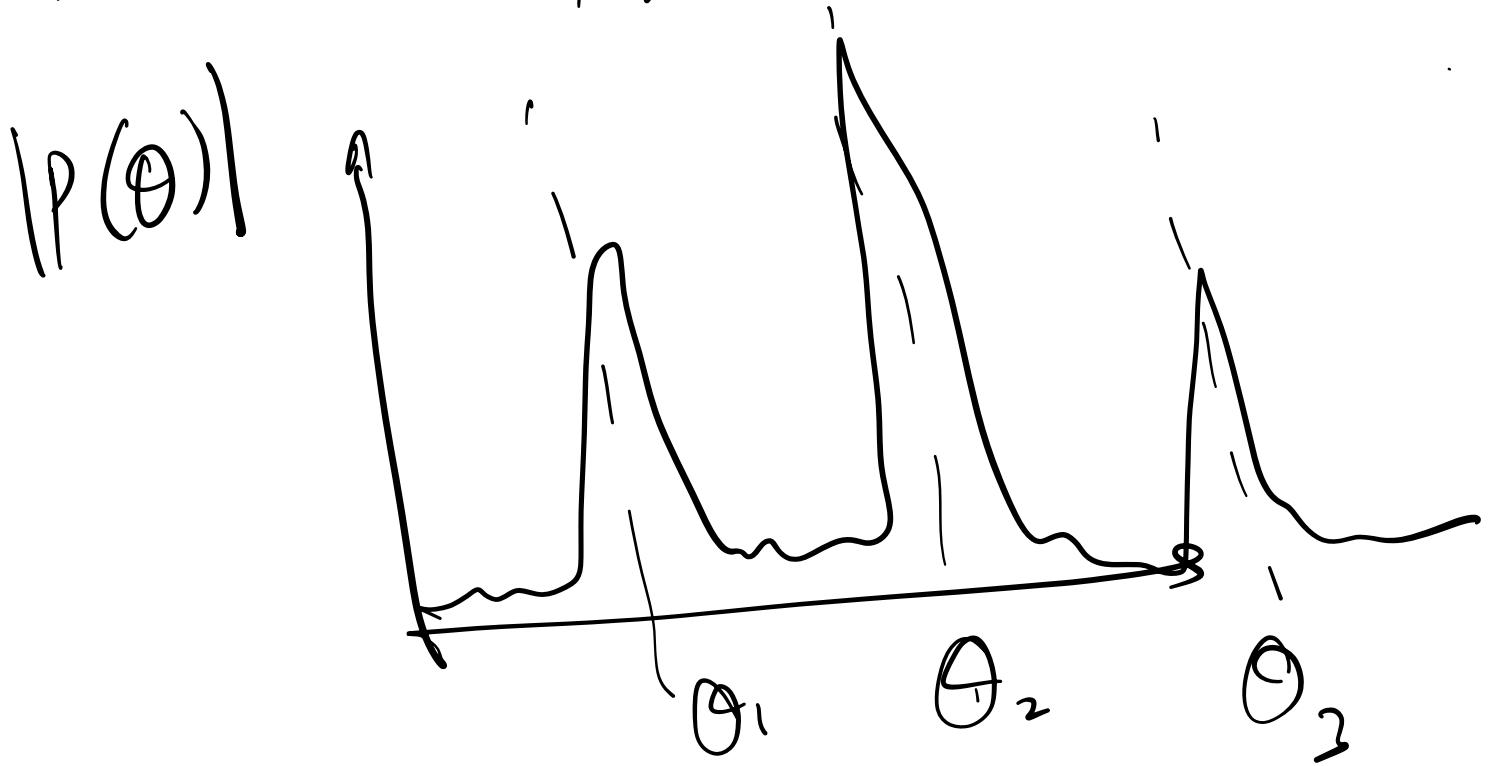
$$\theta_i \neq \theta \Rightarrow \cos \theta_i \neq \cos \theta$$

$$\frac{2\pi}{\lambda} K \cancel{x} [\cos \theta_i - \cos \theta]$$

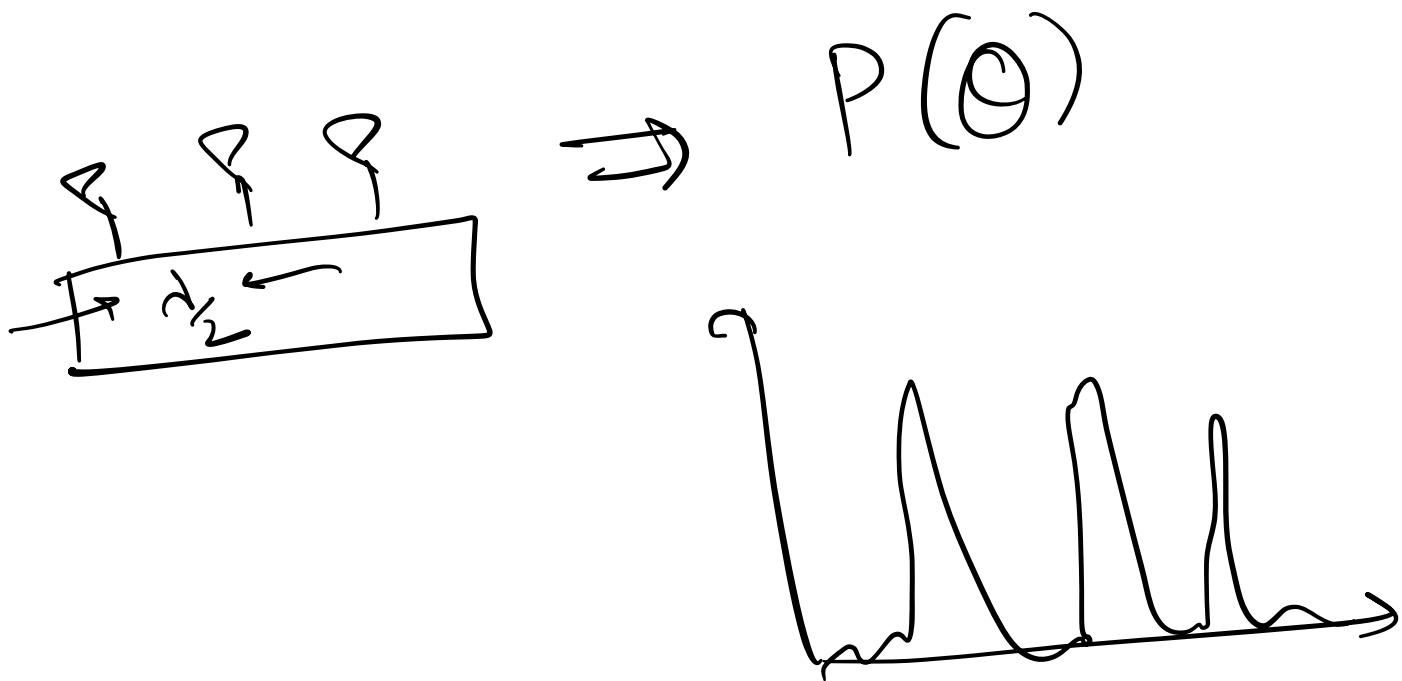


$$P(\theta) = \sum_k h_k e^{+j \frac{2\pi}{\lambda} k x \cos\theta}$$

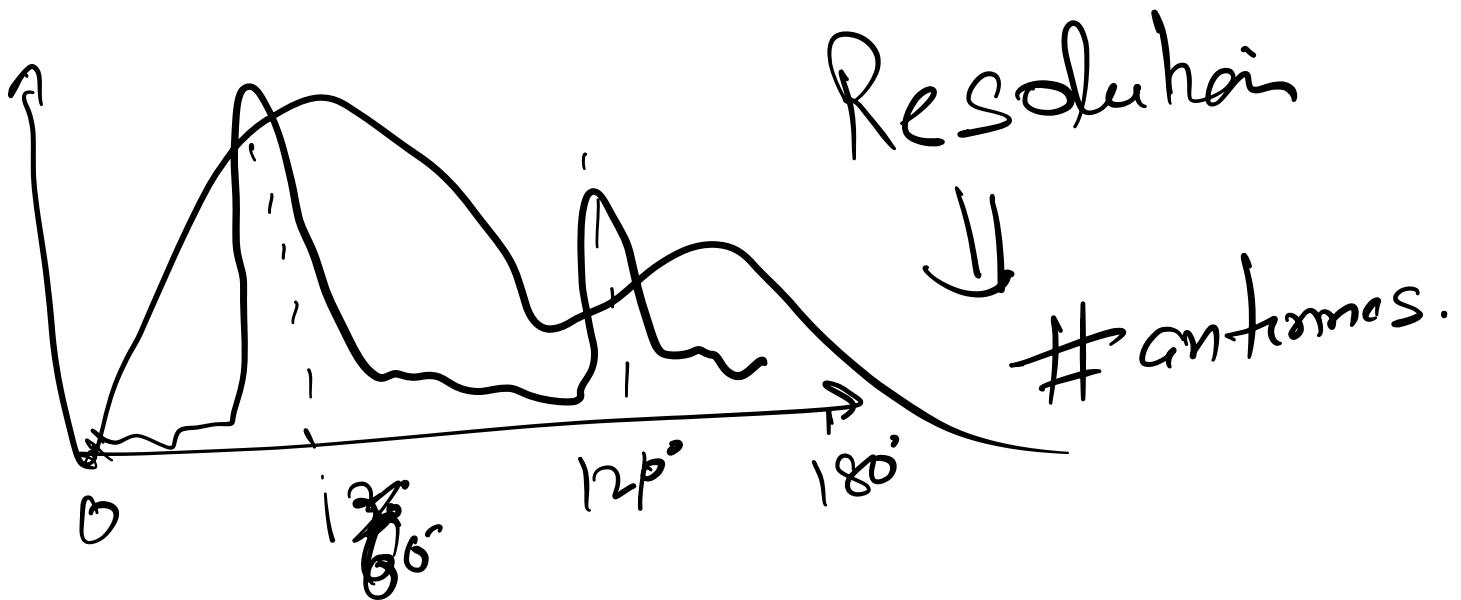
$\omega k \theta)$



Antenna Array

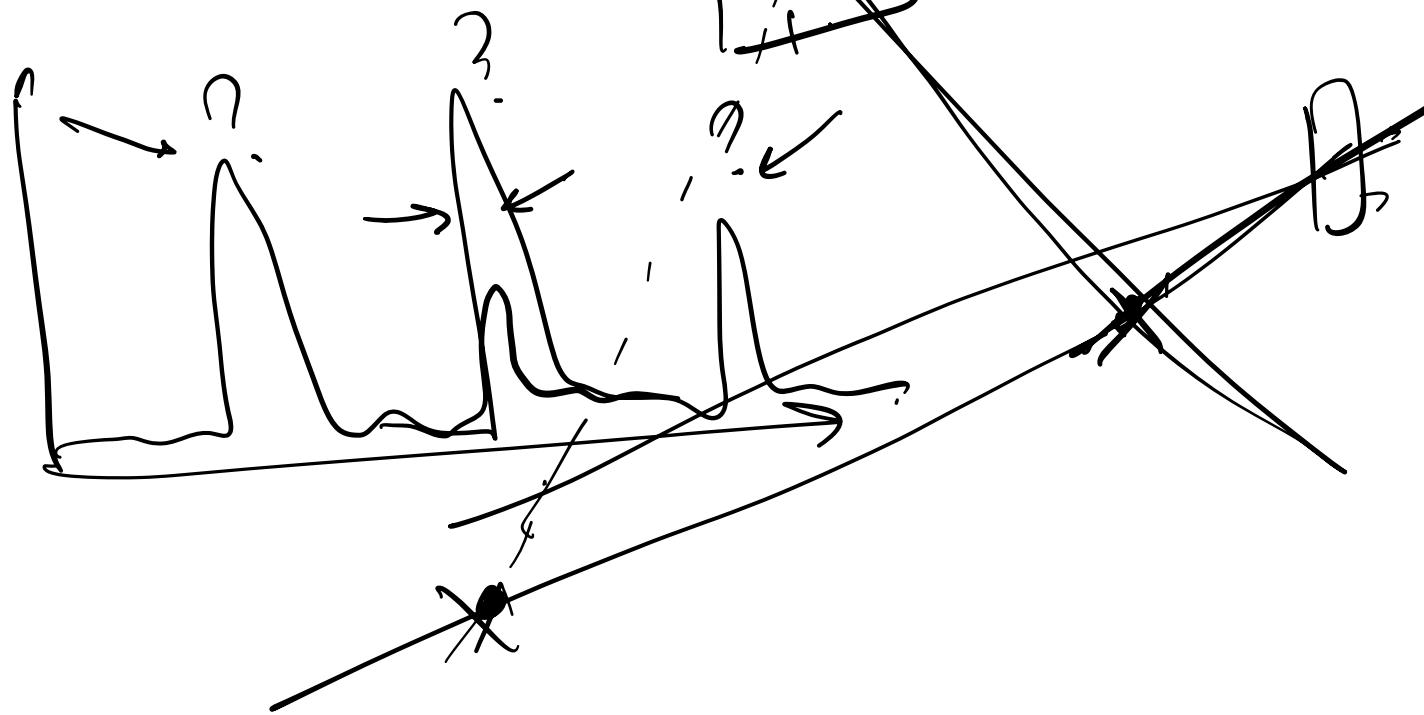
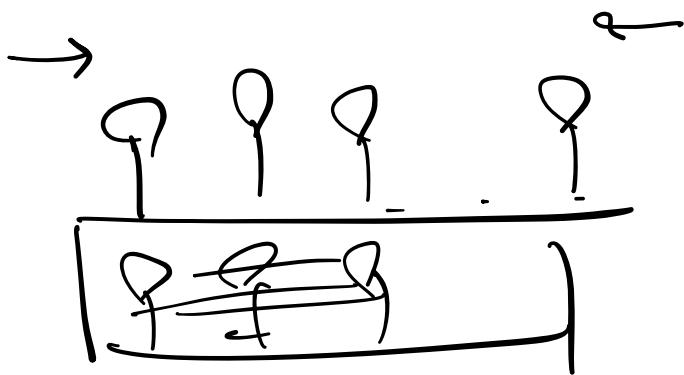
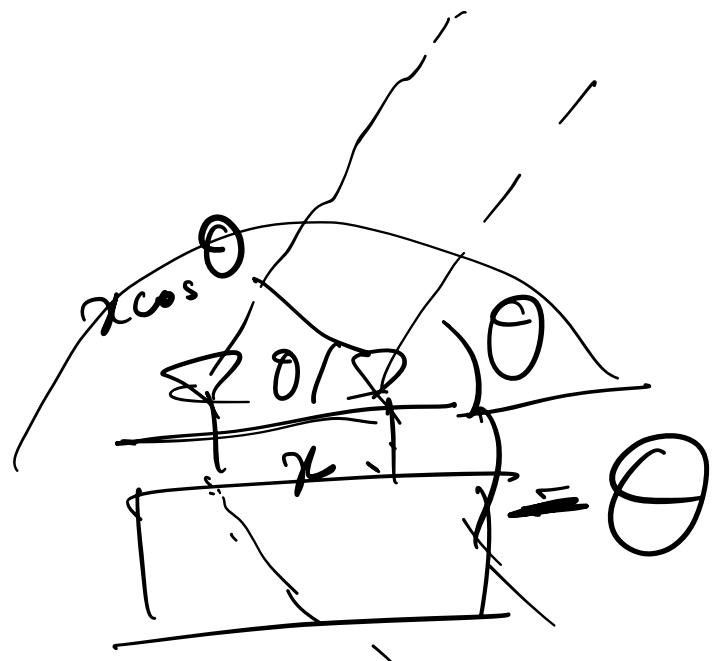


How many antennas do we want?

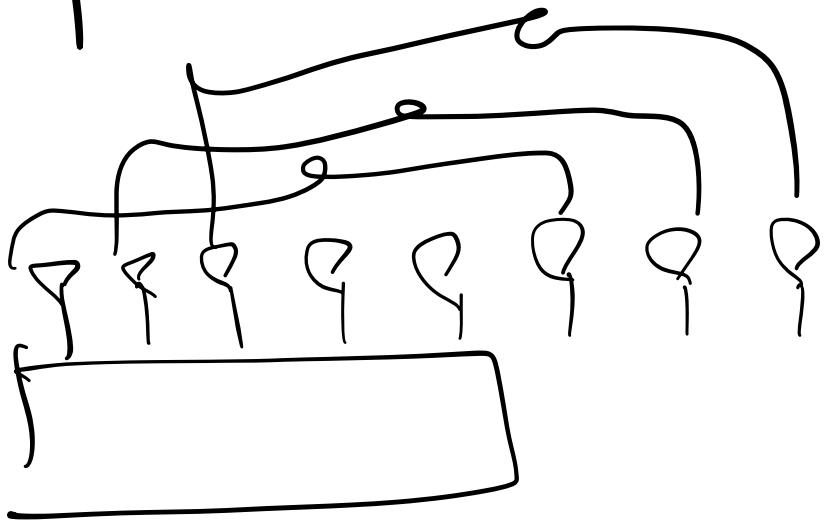


$N \rightarrow \overline{II}$

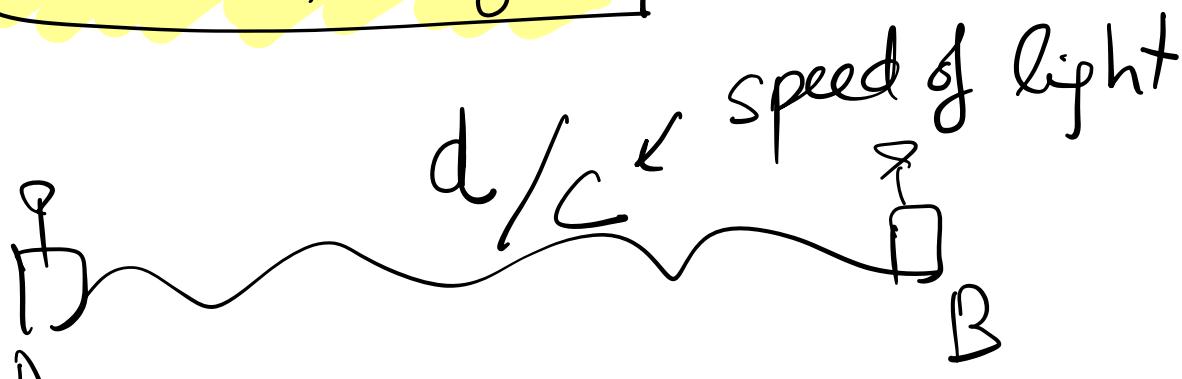
$\frac{-Lb\sin\theta}{N}$



Spatial Smoothing



Time - of - Flight

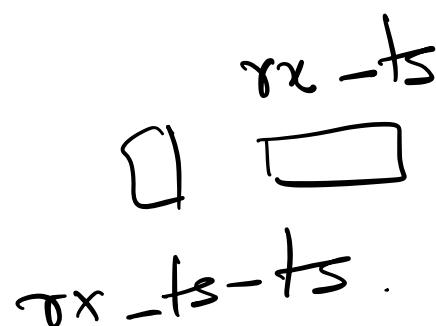


$$t_{\text{of}} = \frac{d}{C} \leftarrow \text{very very large.}$$

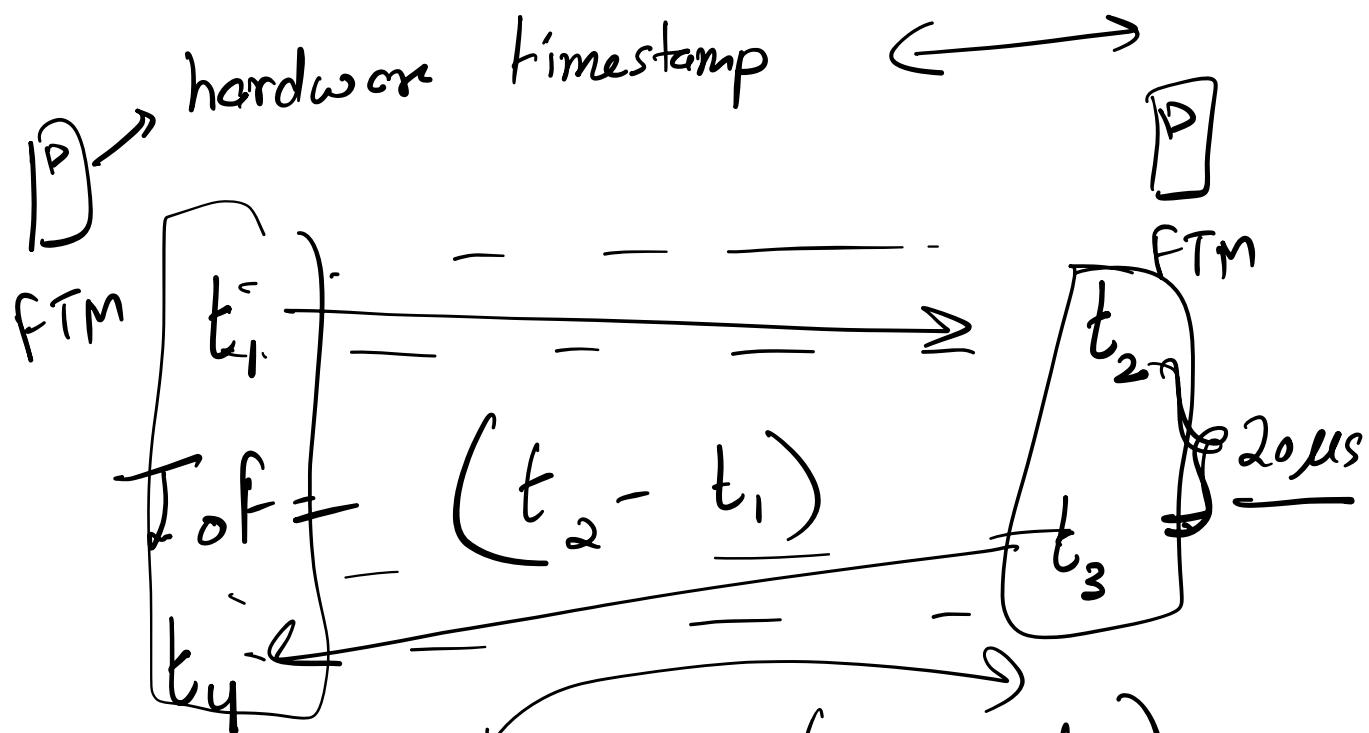
$$1 \text{ ms} \Rightarrow d = 1 \text{ ms} \times 3 \times 10^8 \\ = 3 \times 10^5 = 300 \text{ Km}$$

$$1 \text{ } \mu\text{s} \Rightarrow 300 \text{ m}$$

$$1 \text{ } \underline{\text{ns}} \Rightarrow 30 \text{ cm.}$$



FTM: Fine Time Measurement



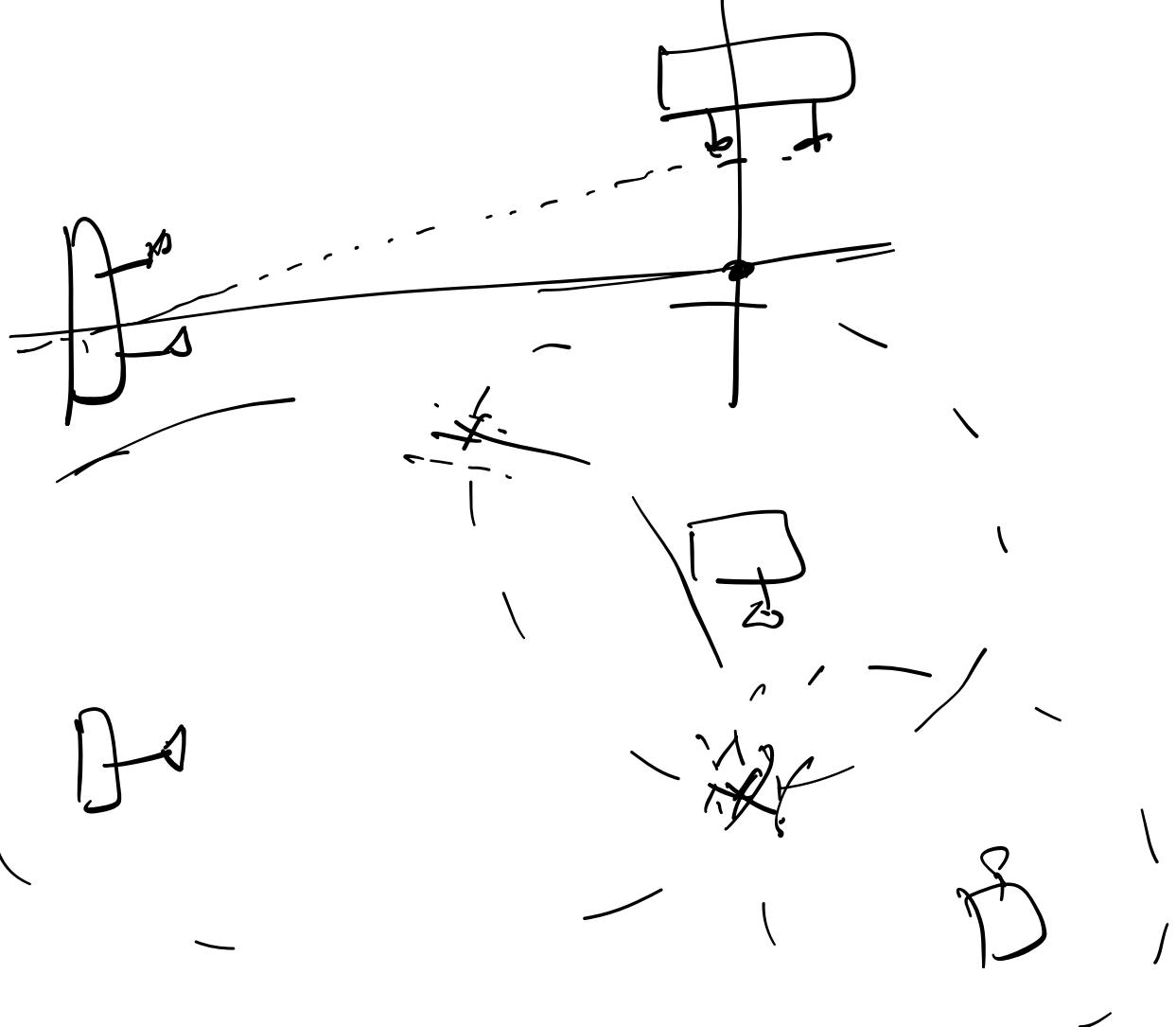
$$T_{OF} = \frac{(t_2 - t_1) + (t_4 - t_3)}{2}$$

$$= \frac{(t_2 - t_3) + (t_4 - t_1)}{2}$$

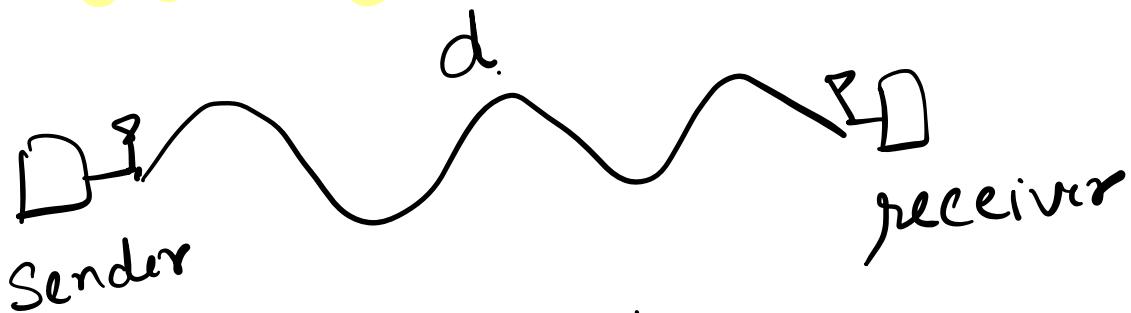
relative
to one device

relative
to one device.

$$\frac{(t_4 - t_1) - (t_3 - t_2)}{2}$$



Distance & Chronos.



$$h = -\frac{2\pi i}{\lambda} d \mod \frac{2\pi i}{\lambda}$$



$$\frac{2h}{2\pi i} = -\frac{d}{\lambda} \mod \frac{\lambda}{2\pi}$$

$$d = -\frac{2h}{\lambda} \mod 2\pi$$

$$\lambda = \cancel{6\text{cm}} \quad d = 1\text{cm}, \quad \cancel{1_a}, \cancel{1_b}, \cancel{1_g}, \\ \cancel{25}, \cancel{26}$$

different value of d . $2.46\text{m} \approx 15\text{cm.}$

$$d = 10\text{cm}, \quad \cancel{25\text{cm}}, \quad 40\text{cm}, \\ \cancel{55\text{cm}}$$

A diagram illustrating wave interference patterns. At the top, a central point emits waves labeled 25 cm . These waves pass through two sets of slits, each with three slits labeled 1 cm apart. The first set of slits is labeled $1\text{ cm}, 13, 19$, and the second set is labeled $25\text{ cm}, 31\text{ cm}, 39\text{ cm}$. The waves from these slits converge at a screen where they interfere constructively at points labeled $d+3\lambda$ and $d+2\lambda$. The distance between the slits and the screen is given by the formula:

$$d = \frac{\lambda_1 h}{2\pi} \text{ mod } \lambda_2 \Rightarrow \{ d \}$$

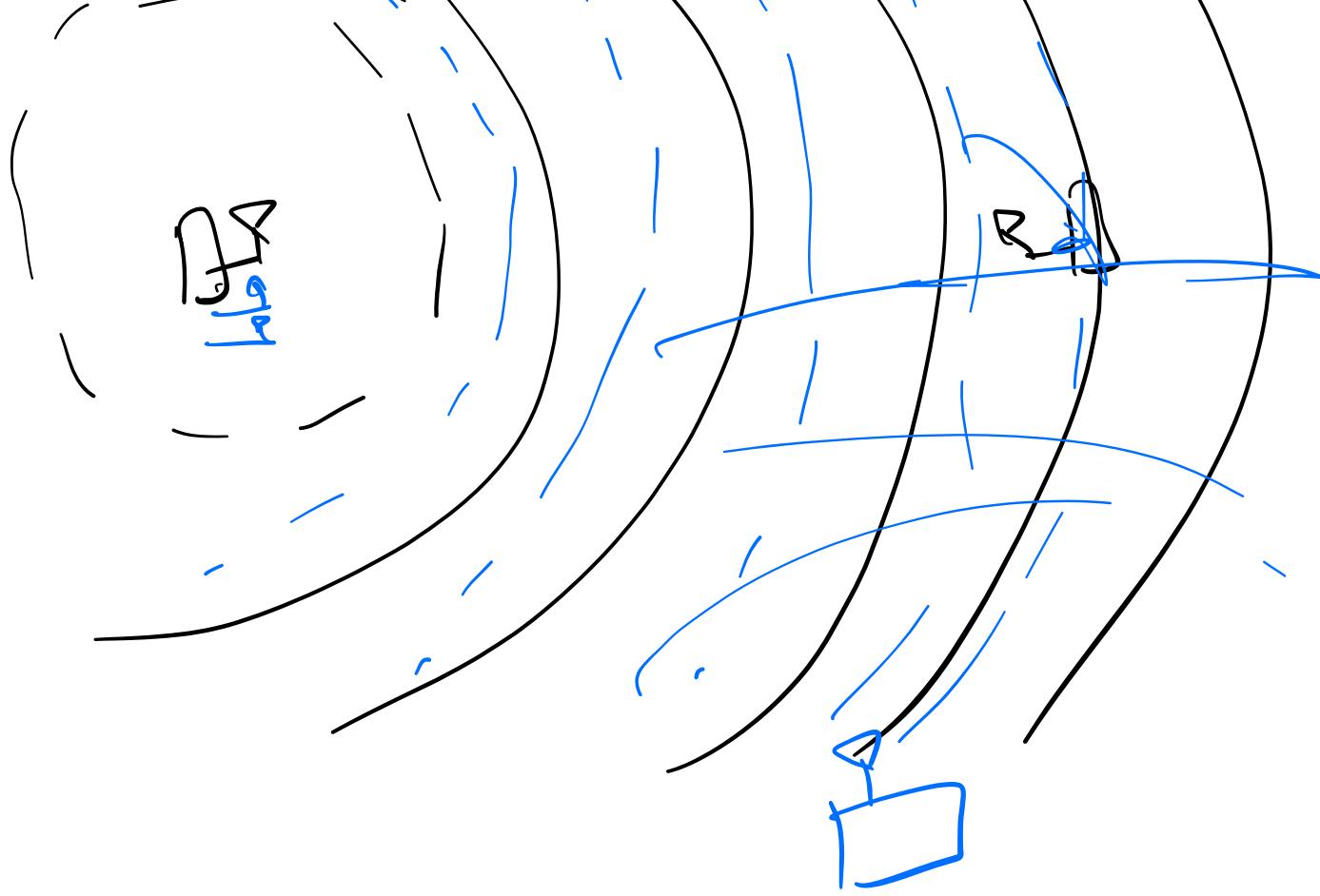
Below this, another formula is shown:

$$d = \frac{\lambda_2 h}{2\pi} \text{ mod } \lambda_2 \Rightarrow \{ d \}$$
 This is followed by a note: "8 cm"

A third formula is shown:

$$d = \frac{\lambda_3 h}{2\pi} \text{ mod } \lambda_3 \Rightarrow \{ d \}$$
 This is followed by a note: "10 cm"

At the bottom, a series of numbers are listed in circles: $5\text{ cm}, 15\text{ cm}, 25\text{ cm}, 35\text{ cm}$.



CFO

→ corrupted phase due to CFO

$$\phi e^{-j2\pi f_c t}$$

$$\phi e^{+j2\pi f_c t}$$

Multipath, CFO , and All That

