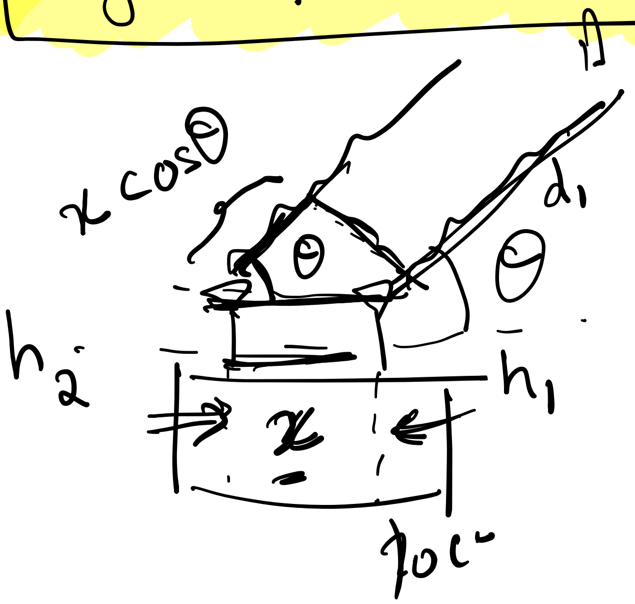
Today

- Angle of Arrival
- Multipath Profile
- Antenna Array
- Time-of-flight
- Distance & Chronos.

Angle of Arrival



$$\angle h_1 = -\frac{2\pi}{\lambda} d_1 \bmod 2\pi$$

$$\angle h_2 = -\frac{2\pi}{\lambda} d_2 \bmod 2\pi$$

$$(d_2 - d_1) = x\cos\theta$$

$$\angle h_1 - \angle h_2 = \frac{2\pi}{\lambda}(d_2 - d_1)$$

$$= \frac{2\pi}{\lambda} x \cos\theta$$

$$\cos\theta = \frac{\lambda}{2\pi}(\angle h_1 - \angle h_2)$$

→ do not capture multipath.

$$\left(\angle h_1 - \angle h_2\right) = \left(\frac{2\pi}{\lambda}\, x \cos\theta\right) \bmod 2\pi$$

mod 2π

-2π, 2π

~~$\cos\theta = \frac{\lambda}{2x}$~~

$-\pi$ π

$$-\pi < \frac{2\pi}{\lambda}\, x\cos\theta < \pi$$

$$-\frac{2\pi}{\lambda}x = -\pi \qquad \frac{2\pi}{\lambda}x = \pi$$

$$x = \frac{\lambda}{2} \qquad x = \frac{\lambda}{2}$$

$d_2 + d_3 > d_1$

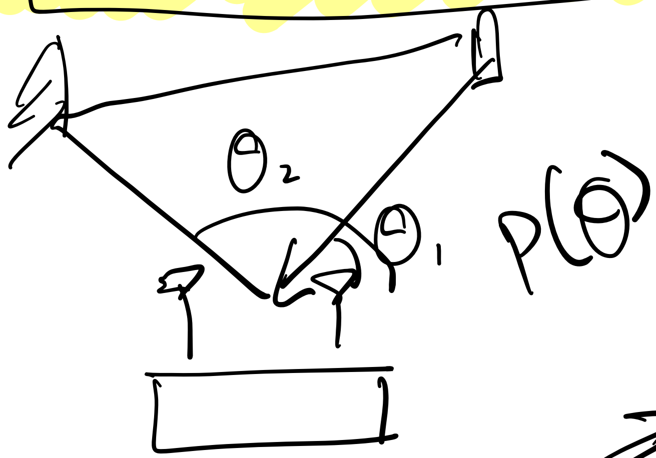
$x = \lambda$

2.4 GHz, → 12 cm

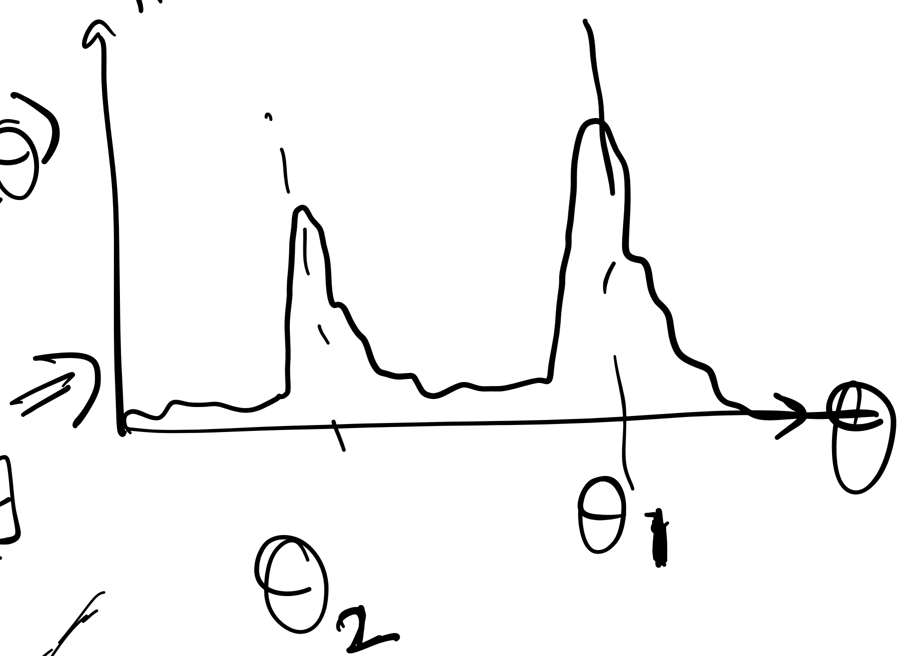
$\lambda_2 = $ [illegible] cm

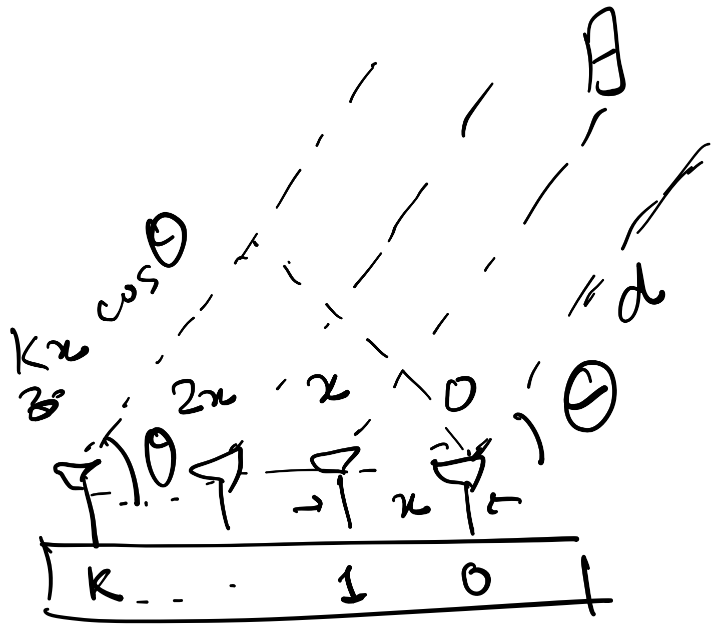
5 GHz $\xrightarrow{6c}$ 3 cm

Multipath Profile



Multipath profile





h_k

h_0

h_1

$\vdots$

h_k

Uniformly Antenna arrays

$$h_k = c\, e^{-j\frac{2\pi}{\lambda}(kx\cos\theta + d)}$$

$$h_1 = c\, e^{-j\frac{2\pi}{\lambda}(x\cos\theta + d)}$$

$$h_2 = c\, e^{-j\frac{2\pi}{\lambda}(2x)\cos\theta + d)}$$

$$h_k = c\, e^{-j\frac{2\pi}{\lambda}(kx\cos\theta + d)}$$

$$h_k = \sum_{i=0}^{N-1} c_i \left(e^{-j\frac{2\pi}{\lambda}(kx\cos\theta_i + d_i)}\right)$$

10 antennes, 3paths.

$$P(\theta) = \sum_k h_k e^{+j\frac{2\pi}{\lambda}(kx\cos\theta)}$$

$$h_k = c_1 e^{-j\frac{2\pi}{\lambda}(kx\cos\theta_1 + d_1)}$$

$$P(\theta) = \sum_{k} c_1 e^{-j\frac{2\pi}{d}(kx\cos\theta_1 + d_1)}$$

$$e^{j\frac{2\pi}{d}kx\cos\theta}$$

$$= c_1 e^{-j\frac{2\pi}{d}d_1} \sum_{k} e^{-j\frac{2\pi}{d}kx[\cos\theta_1 - \cos\theta]}$$

$$\underbrace{e^{-j\frac{2\pi}{d}kx[\cos\theta_1 - \cos\theta]}}_{=1}$$

$$\theta_1 = \theta \Rightarrow \cos\theta_1 = \cos\theta \Rightarrow 0$$

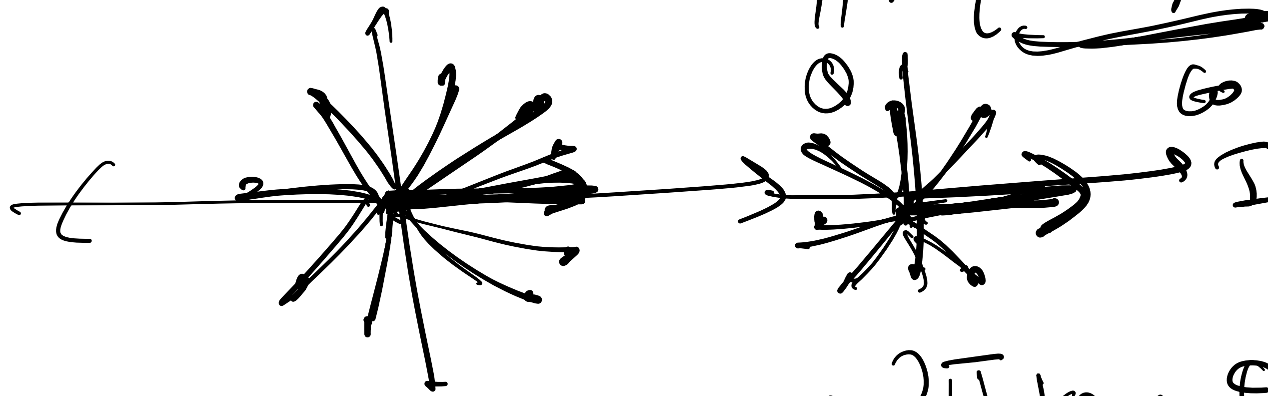
$$P(\theta) = k\, c_1 e^{-j\frac{2\pi}{d}d_1}$$

$$\theta_1 \neq \theta \Rightarrow \cos\theta_1 \neq \cos\theta$$

$$\frac{2\pi}{d} k \frac{x}{x} [\cos\theta_1 - \cos\theta]$$

$$\Rightarrow \sum e^{-j \pi k [\cos\theta_1 - \cos\theta]}$$

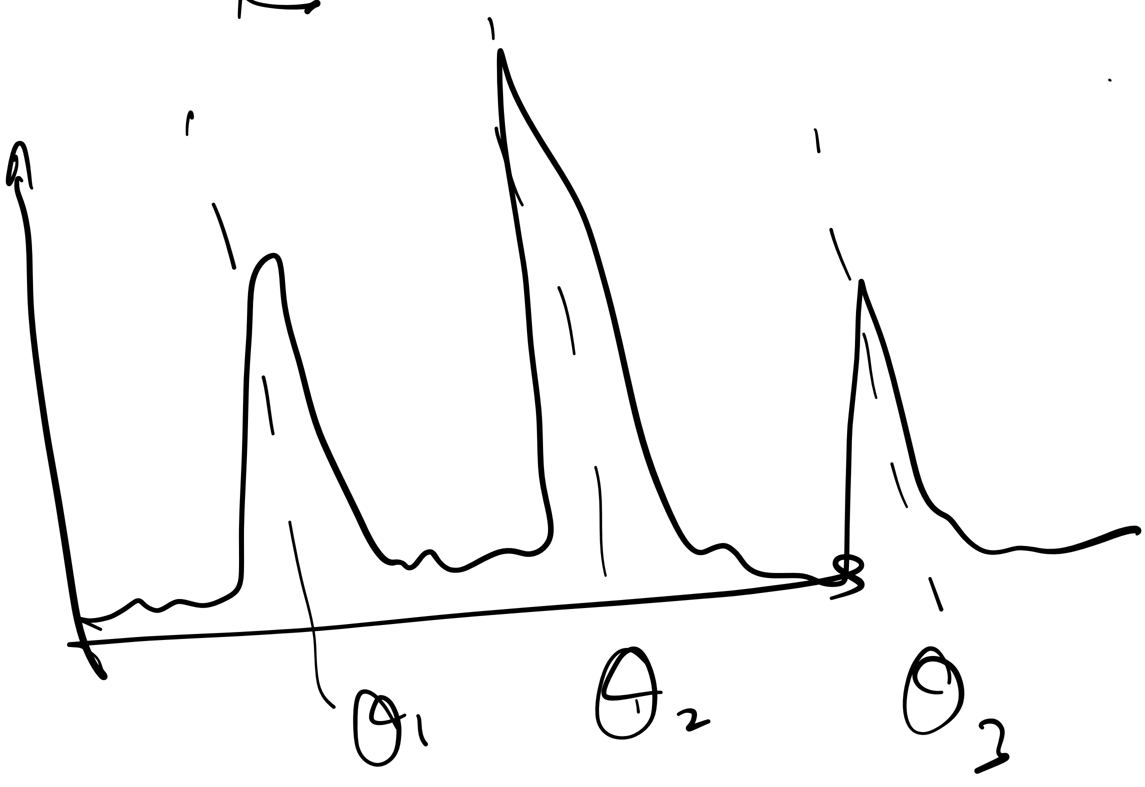
$$\pi k [\cos\theta_1 - \cos\theta]$$



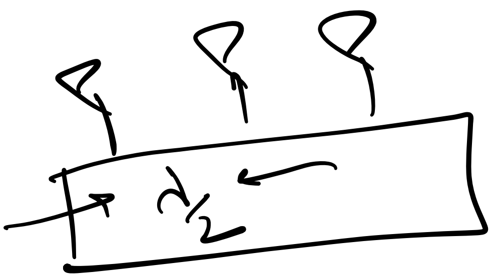
$$P(\theta) = \sum_k h_k e^{+j \frac{2\pi}{\lambda} k x \cos\theta}$$

$\cos(-\theta)$

$|P(\theta)|$

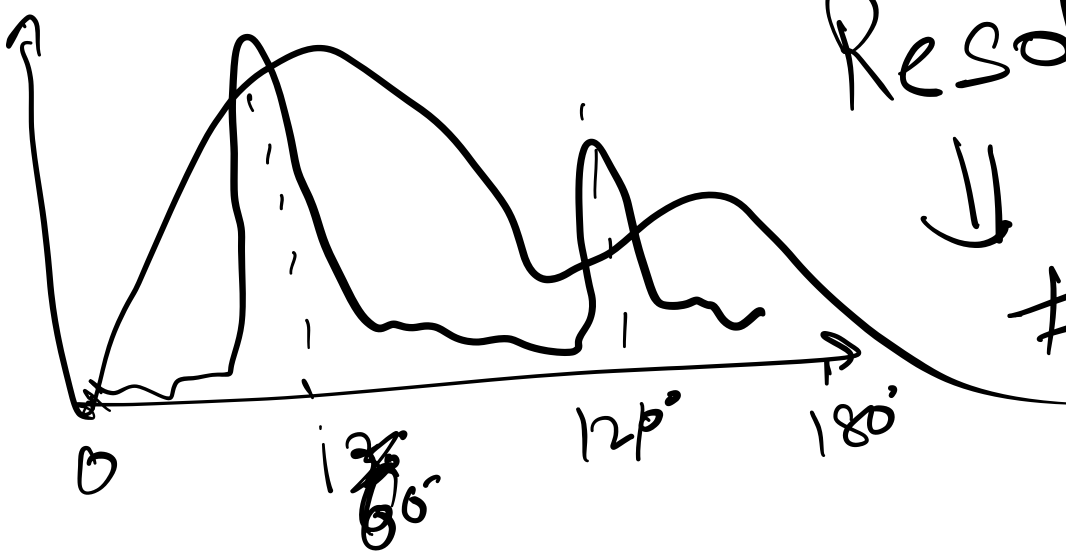


Antenna Array



$\Rightarrow$ $P(\theta)$

How many antennas do we want?



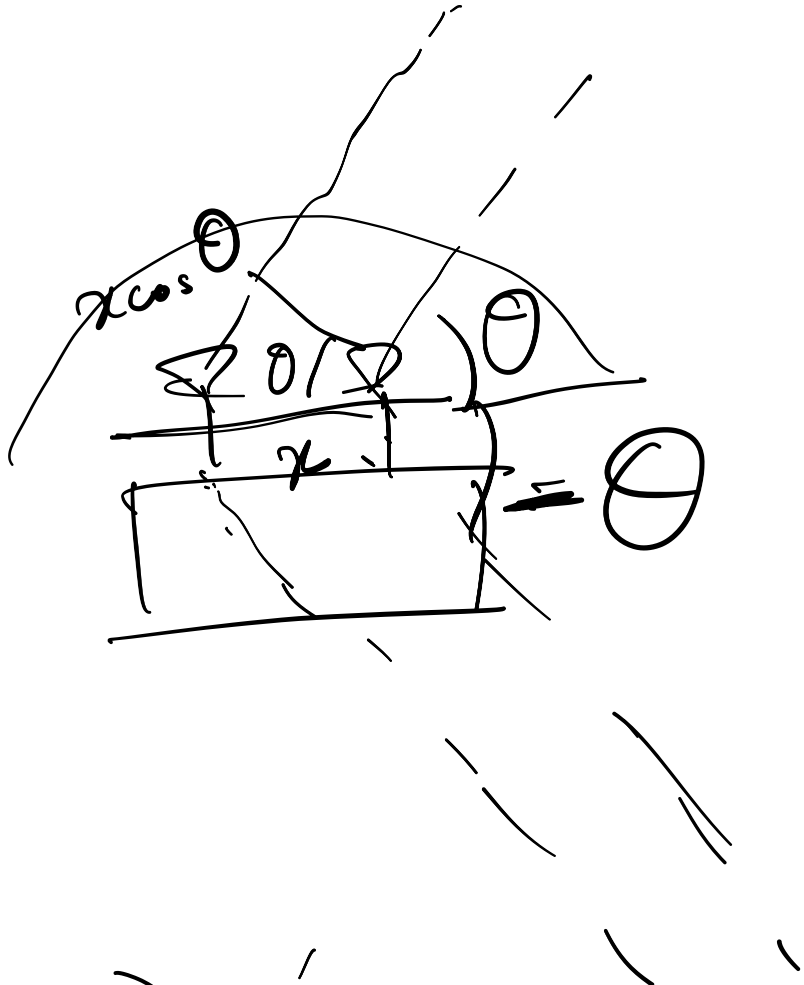
Resolution

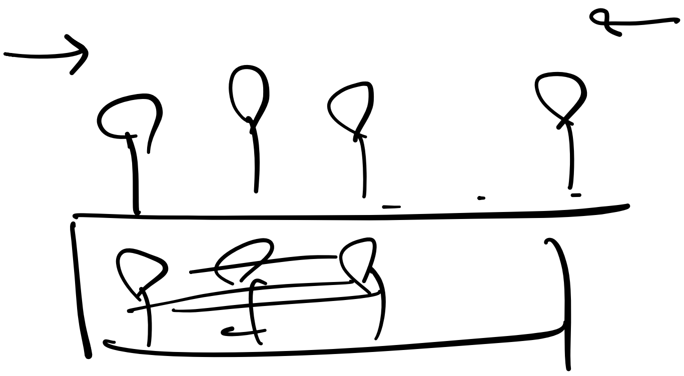
$\Downarrow$

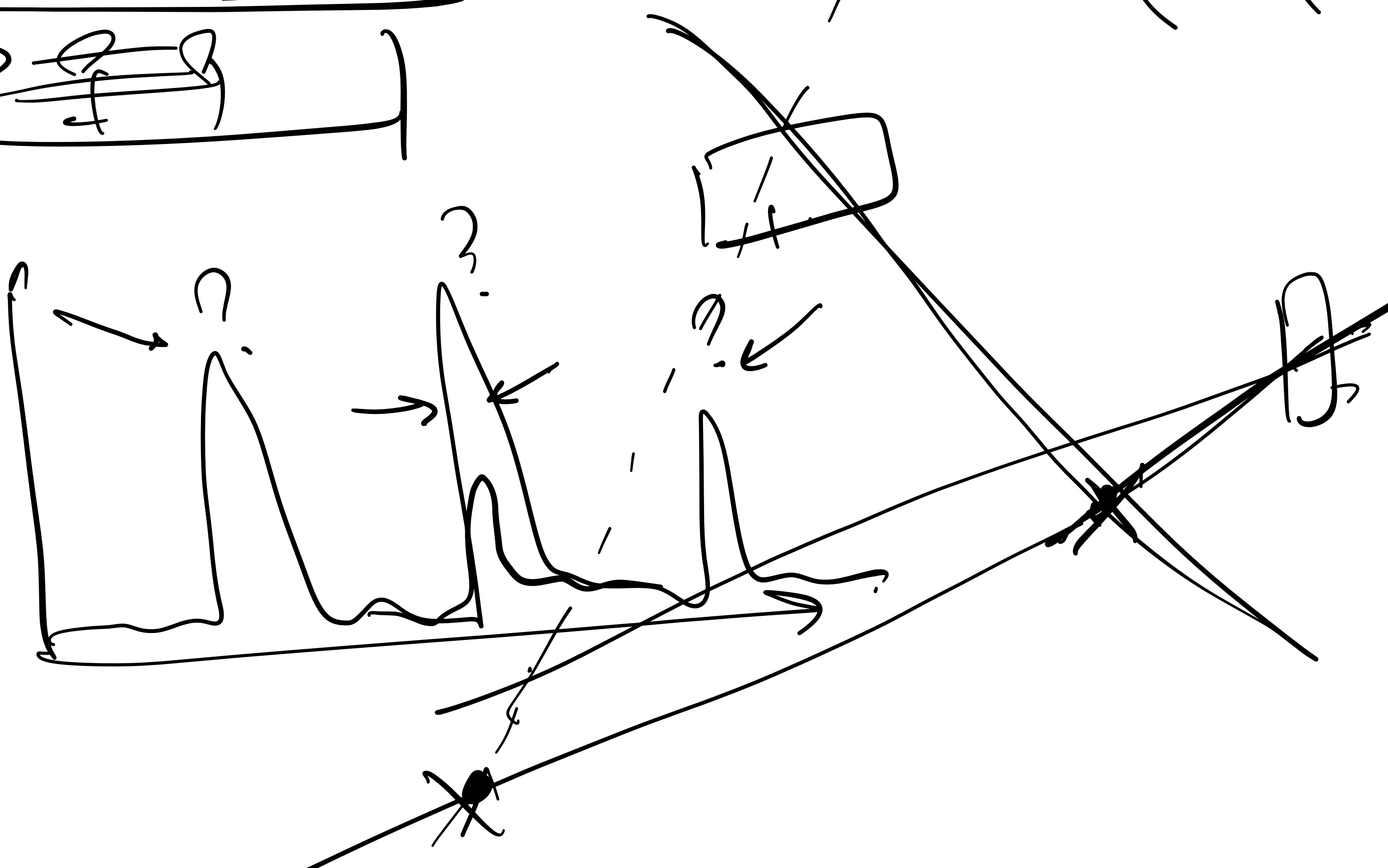
antennas.

$N \Rightarrow \frac{\pi}{N}$

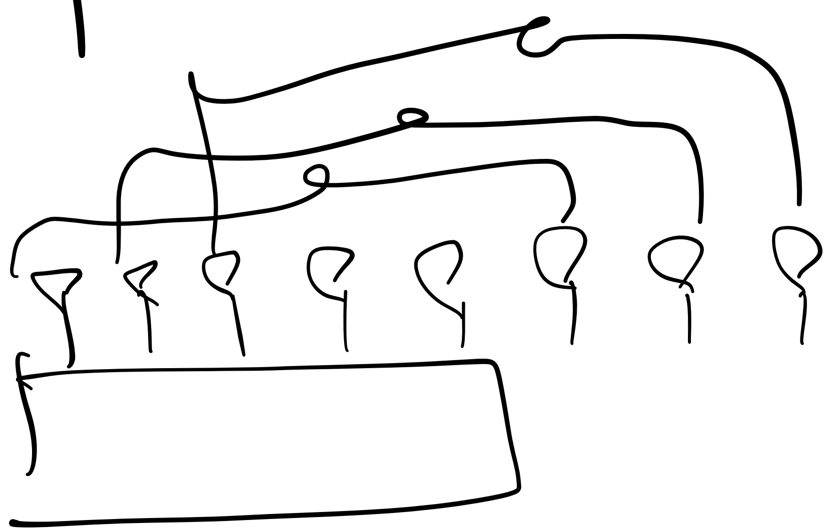
$-\frac{\cos\theta}{N}$



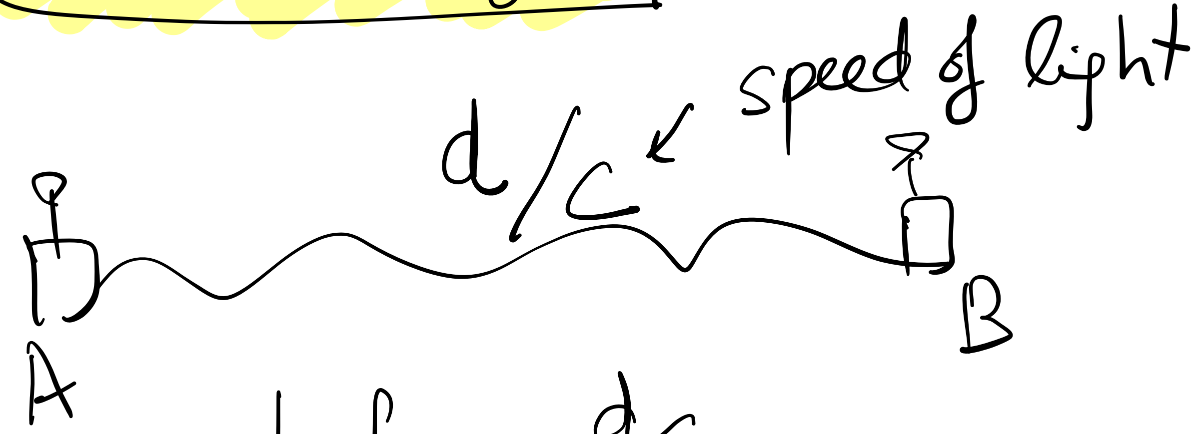




Spatial Smoothing



Time-of-Flight



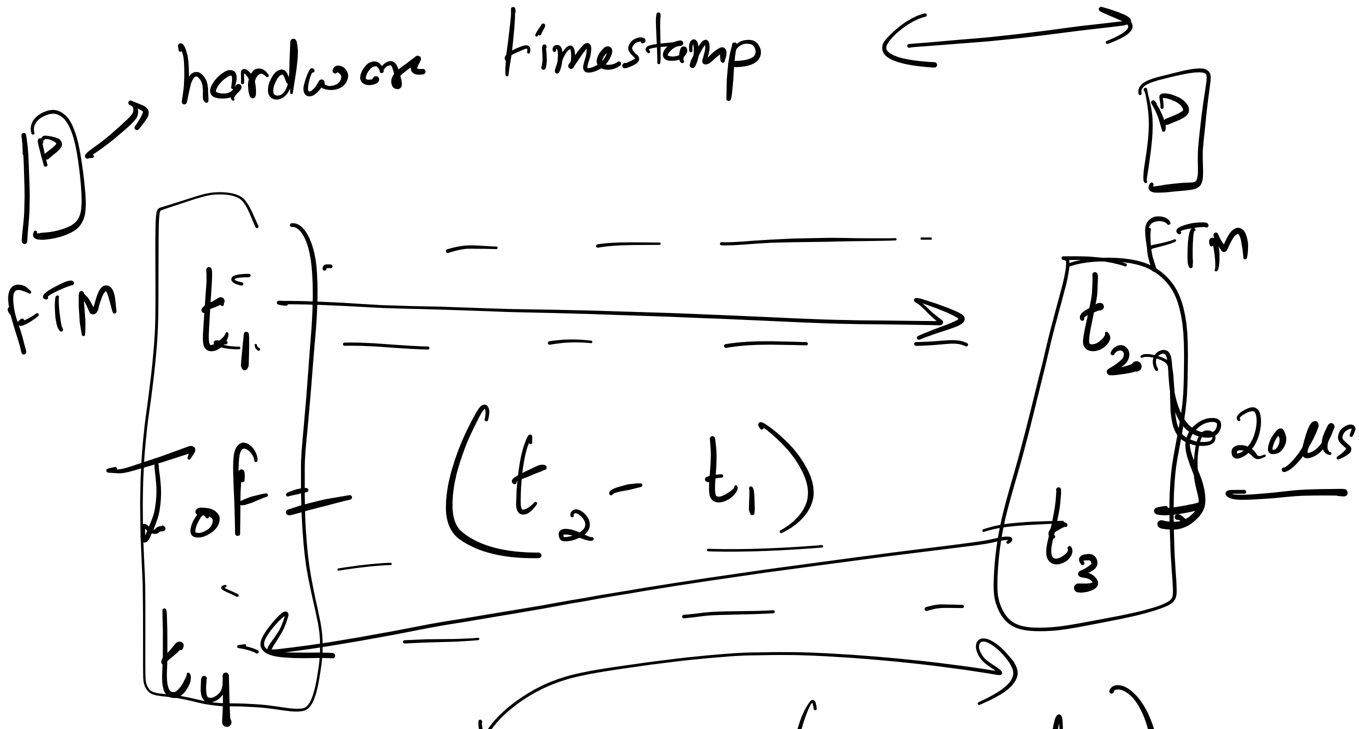
$$ToF = d/c$$ ← veryvery large.

1 ms ⇒ $d = 1ms \times 3\times10^8$
$= 3\times10^5 = 300$ Km

1 μs ⇒ 300 m

1 ns ⇒ 30 cm.

FTM: Fine Time Measurements



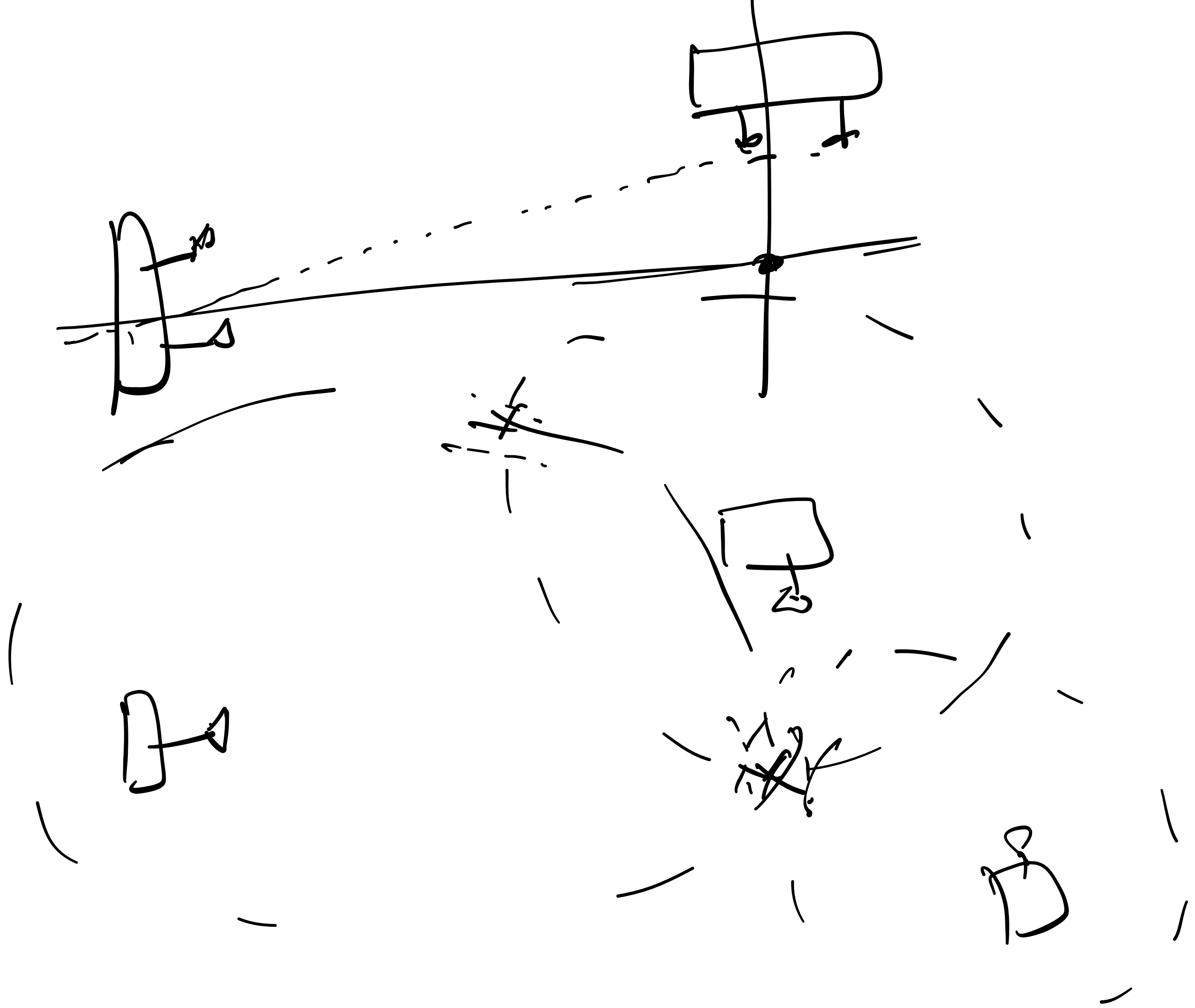
$$Tof = \frac{(t_2 - t_1) + (t_4 - t_3)}{2}$$

$$= \frac{(t_2 - t_3) + (t_4 - t_1)}{2}$$

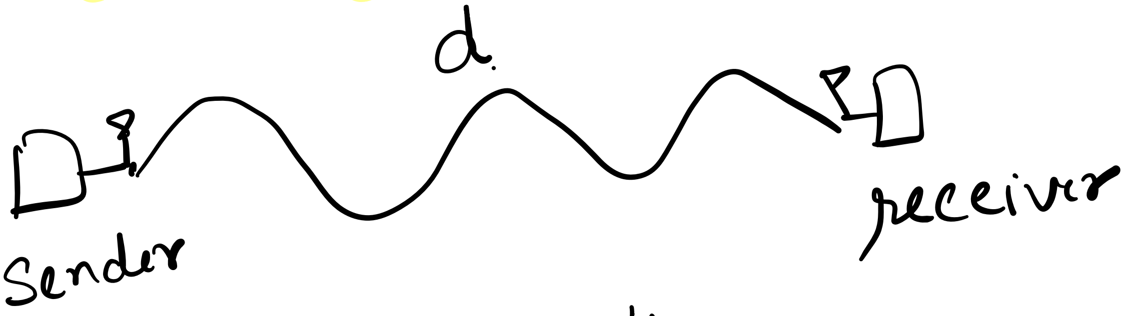
relative to one device

relative to one devi.

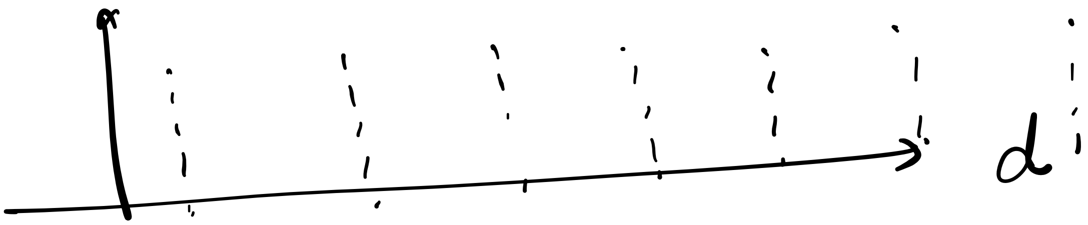
$$\frac{(t_4 - t_1) - (t_3 - t_2)}{2}$$



Distance & Chronos.



$$\angle h = -\frac{2\pi}{\lambda} d \mod 2\pi$$



$$\lambda \frac{\angle h}{2\pi} = -d \mod \lambda$$

$$d = -\frac{\lambda \angle h}{2\pi} \mod 2\pi$$

$\lambda = 6$ cm $\quad d = 1$ cm, 7c, 13c, 19, 25, 26c

different value of λ. 2.4GHz ≈ 15 cm.

d = 10c, 25c, 40c, 55c

25cm,   10km

1cm

$0 < d < \lambda \leq 6\text{cm}$.

$d+3\lambda$

$d+\lambda$ $d+2\lambda$

7cm, 7c, 13, 19, 25cm, 31cm, 37cm,

$$d = \frac{\lambda_1 \angle h}{2\pi} \bmod \lambda_1 \Rightarrow \{d\}$$

$$d = \frac{\lambda_2 \angle h}{2\pi} \bmod \lambda_2 \Rightarrow \{d\}$$

" 8cm

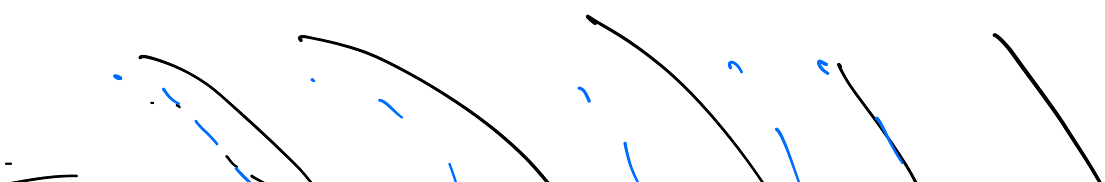
⇒ 1cm, 9cm, 17cm, 25cm, 33c..

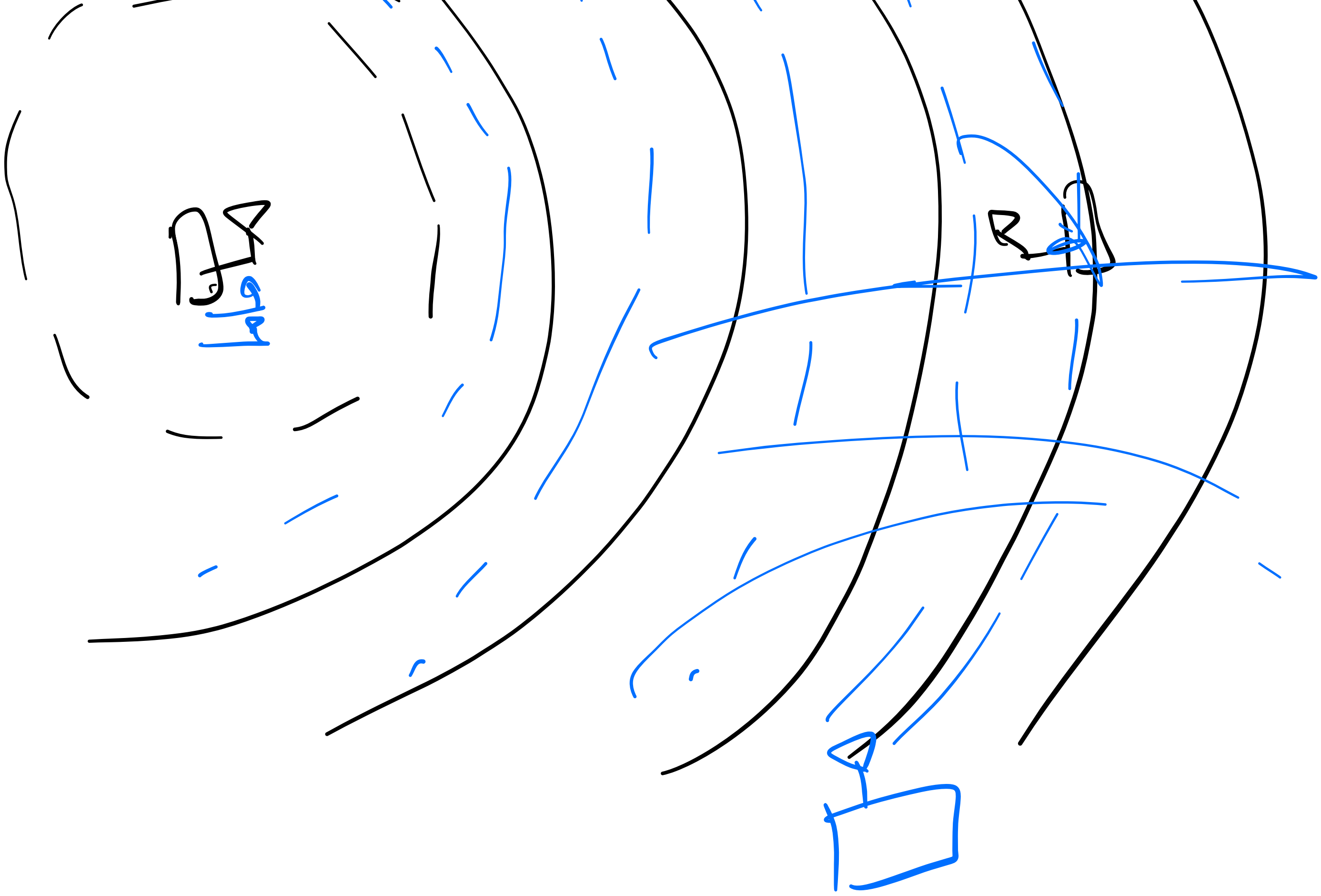
$$d = \frac{\lambda_3 \angle h}{2\pi} \bmod \lambda_3 \Rightarrow \{d\}$$

⇓

10cm

5cm, 15cm, 25cm, 35cm;





CFO → corrupted phase due to CFO

$\phi \, e^{-j2\pi f_c t}$ $\quad\quad$ $\phi \, e^{+j2\pi f_c t}$

Multipath, CFO, and All That